# Online Identification of Power System Oscillation Modes Based on Mode Shape Matching

Chengfei Wang, Chongtao Li, Guihong Wu, Guiyuan Li, Zhengchun Du, Member, IEEE

Abstract—This paper proposes a novel online mode identification formula, which firstly applies the results from modal analysis to online mode identification and so overcomes the defect of measurement-based methods that results from them cannot be directly used to damp oscillations. In this formula, a Prony-based multi-signal processing (PMP) method is developed to estimate the mode shape from ringdown data and a mode shape matching method is firstly introduced for finding out which modes are activated in the real-time oscillation of a large-scale power system. Simulation studies based on China Southern Power Grid have been carried out to verify the correctness and applicability of the proposed formula and methods.

*Index Terms*—Online mode identification, mode shape estimation, Arnoldi method, mode shape matching.

### I. INTRODUCTION

**P**OWER system small signal stability problem is one of the most serious threats to system stability [1]. To damp oscillations caused by this problem, a precise and real-time identification of dominant modes is the basic requirement for system operators to adopt emergency oscillation control.

Generally, two kinds of methods have been proposed to identify dominant modes of a power system, the traditional modal-based methods [2]-[4] and the measurement-based methods [5]-[13] using PMU data. Even though modal analysis can obtain plenty of control information, without real-

time monitoring, operators can't identify which modes are activated in the real-time oscillation.

To identify dominant modes online, methods such as Prony analysis [5], Matrix Pencil method [6], N4SID method [7], and RLS method [8]-[9] have been proposed. As for the mode shape estimation, methods such as Spectral-based method [10]-[11] and Transfer Function method [12]-[13] are also proposed. However, though measurement-based methods can identify dominant modes in near real time, lack of participation factors and sensitivities makes them unable to be directly applied to emergency oscillation control.

Set the the equilibrium point of modal analysis as stable operating point after fault. Then theoretically, we can find the corresponding mode of  $\lambda_m$  in the results of model-based methods, which is defined as the activated mode. The activated mode carries information of real-time oscillation and system model, so it can be used to damp the oscillation.

To realize online mode identification, the novel online mode identification formula is proposed in this paper. Firstly, obtain the dominant modes  $\lambda_m$  and corresponding mode shapes from ringdown data of the real-time oscillation. Second, based on the system model, Arnoldi method is used to search the closest modes of  $\lambda_m$ . Finally, the activated modes should be identified by mode shape matching.

Just as shown in Case B of Fig. 1, there are many similar modes in the results from model-based method.  $\lambda_m$  may correspond to one of these similar modes or may be a superposition of them. In such case, we cannot figure out which modes are activated according to only the differences between modes. To solve this problem, the mode shape matching method is firstly proposed in this paper.

Once the activated modes are identified by the formula, operators can take measures of control in near real time [14].

The remainder of this paper is organized as follows. Section II and III present the PMP method and modal analysis. The mode shape matching method is proposed and explained in

This work was supported by Key Project of Smart Grid Technology and Equipment of National Key Research and Development Plan of China (No.2016YFB0900600).

C. W., C. L., G. W. and Z. D. are with School of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, China (e-mail: xjtu568937435@stu. xjtu.edu.cn, ctali@163.com, wghwest@stu.xjtu.edu.cn, zcdu@xjtu.edu.cn).

G. L. is with State Key Laboratory of HVDC (Electric Power Research Institute, China Southern Power Grid), Guangzhou 510663, China (e-mail: liguiyuan@stu.xjtu.edu.cn)

Section IV. Simulations are implemented in Section V to validate the correctness and applicability of the proposed formula and methods. Conclusions are shown in Section VI.



Fig. 1. Different cases in finding out the activated modes.

#### II. THE PMP METHOD

In this section, a new Prony-based multi-signal processing (PMP) method is proposed to obtain the dominant modes and corresponding mode shapes from the ringdown data.

If a system can be described by a linear state space model, its homogeneous responses are a sum of exponentially damped sinusoidal signals, which are also known as the ringdown data:

$$\tilde{y}[j] = \sum_{i=1}^{n} c_i z_i^j, z_i = e^{\lambda_i \Delta t} \quad j = k, k+1, \dots, k+N-1$$
(1)

where  $\tilde{y}[j]$  is the pure ringdown data at time  $j\Delta t$ ,  $\lambda_i$  is the mode to be estimated,  $\Delta t$  is the sampling interval, *n* is the number of eigenvalues, *N* is the number of measurement data in a sample window,  $c_i$  is the amplitude of the *i*-th mode.

Define  $\hat{y}[j]$  as the measurement ringdown data described by (1). Note that it contains measurement and process noise in addition to the pure ringdown signal. To filter out noise, the number of measurement data in a sample window *N* is usually selected to be greater than 2n [5].

Firstly, apply the least square method to solve the following equations to calculate the  $\theta[k] = [a_1, a_2, \dots, a_n]^T$ 

$$\hat{\boldsymbol{y}}_{multi}[k] = \boldsymbol{H}_{multi}[k]\boldsymbol{\theta}[k] + \boldsymbol{e}_{multi}[k]$$
(2)

where

$$\begin{cases} \hat{\boldsymbol{y}}_{multi}[k] = [\hat{\boldsymbol{y}}_{1}[k] \quad \hat{\boldsymbol{y}}_{2}[k] \quad \cdots \quad \hat{\boldsymbol{y}}_{m}[k]] \\ \hat{\boldsymbol{y}}_{i}[k] = [\hat{\boldsymbol{y}}_{i}[k+n] \quad \hat{\boldsymbol{y}}_{i}[k+n+1] \quad \cdots \quad \hat{\boldsymbol{y}}_{i}[k+N-1]]^{T} \end{cases}$$
(3)
$$\begin{cases} \boldsymbol{H}_{multi}[k] = [\boldsymbol{H}_{1}[k], \boldsymbol{H}_{2}[k], \cdots, \boldsymbol{H}_{m}[k]] \\ \boldsymbol{H}_{i}[k] = [\hat{\boldsymbol{y}}_{i}^{T}[k-1] \quad \hat{\boldsymbol{y}}_{i}^{T}[k-2] \quad \cdots \quad \hat{\boldsymbol{y}}_{i}^{T}[k-n]] \end{cases}$$
(4)

are the measurement ringdown data of different state variables, and *m* is the number of measured state variables.

$$\begin{cases} \boldsymbol{e}_{multi}[k] = \begin{bmatrix} \boldsymbol{e}_1[k] & \boldsymbol{e}_2[k] & \cdots & \boldsymbol{e}_m[k] \end{bmatrix} \\ \boldsymbol{e}_i[k] = \begin{bmatrix} \boldsymbol{e}_i[k+n] & \boldsymbol{e}_i[k+n+1] & \cdots & \boldsymbol{e}_i[k+N-1] \end{bmatrix}^T \end{cases}$$
(5)

are the error vectors, where  $e_i[k]$  represents process noise.

Then the least-square problem can be described as

$$\boldsymbol{\theta}[k] = \min \begin{bmatrix} (\hat{\boldsymbol{y}}_{multi}[k] - \boldsymbol{H}_{multi}[k]\boldsymbol{\theta}[k])^{T} \\ \cdot \boldsymbol{\Lambda} \cdot (\hat{\boldsymbol{y}}_{multi}[k] - \boldsymbol{H}_{multi}[k]\boldsymbol{\theta}[k]) \end{bmatrix}$$
(6)

where  $\Lambda$  is composed of the forgetting factor, a positive constant slightly smaller than or equal to 1 [15].

After obtaining the coefficients of characteristic polynomial  $\theta[k]$  by solving (6), modes can be easily gotten by solving the polynomial

$$\hat{z}^{n} - [\hat{a}_{1}\hat{z}^{n-1} + \hat{a}_{2}\hat{z}^{n-2} + \dots + \hat{a}_{n}\hat{z}^{0}] = 0 \quad \hat{s}_{i} = \frac{1}{\Delta t}\ln(\hat{z}_{i})$$
(7)

Then the Prony coefficient can be estimated by the least square method

$$\mathbf{Z} \cdot \mathbf{C} = \mathbf{Y} \tag{8}$$

$$\mathbf{Z} = \begin{bmatrix} \hat{z}_{1}^{0} & \hat{z}_{2}^{0} & \cdots & \hat{z}_{n}^{0} \\ \hat{z}_{1}^{1} & \hat{z}_{2}^{1} & \cdots & \hat{z}_{n}^{1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{z}_{1}^{N-1} & \hat{z}_{2}^{N-1} & \cdots & \hat{z}_{n}^{N-n-1} \end{bmatrix} \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nm} \end{bmatrix}$$
(9)  
$$\mathbf{Y} = \begin{bmatrix} \hat{y}_{1}[k] & \hat{y}_{2}[k] & \cdots & \hat{y}_{m}[k] \\ \hat{y}_{1}[k+1] & \hat{y}_{2}[k+1] & \cdots & \hat{y}_{m}[k+1] \\ \vdots & \vdots & \ddots & \vdots \\ \hat{y}_{1}[k+N-1] & \hat{y}_{2}[k+N-1] & \cdots & \hat{y}_{m}[k+N-1] \end{bmatrix}$$
(10)

Set the state variables  $\Delta x_l$  (*l*=1,2,...,*m*) as the measured data. Then (1) can be reconstructed as

$$\Delta x_{l}[j] = c_{1l}e^{\lambda_{l}j\Delta t} + c_{2l}e^{\lambda_{2}j\Delta t} + \dots + c_{nl}e^{\lambda_{n}j\Delta t} \quad (l = 1, 2, \dots, m) \quad (11)$$

Notice that in modal analysis, the time domain response of state variables is [4]

$$\Delta x_{l}(t) = k_{1} v_{l,1} e^{\lambda_{1} t} + k_{2} v_{l,2} e^{\lambda_{2} t} + \dots + k_{n} v_{l,n} e^{\lambda_{n} t}$$
(12)

Considering that the coefficients of exponential term in (11) and (12) should be equal respectively, it can be seen that for a certain mode  $\lambda_i$ ,  $c_{il} = k_i v_{li}$ . Further, for different state variables  $x_1, x_2, \dots, x_m$ , we can get

$$\boldsymbol{c}_i = \boldsymbol{k}_i \cdot \boldsymbol{v}_i \tag{13}$$

where  $\boldsymbol{c}_{i} = [c_{i1}, c_{i2}, \dots, c_{im}]^{T}$ ,  $\boldsymbol{v}_{i} = [v_{1i}, v_{2i}, \dots, v_{mi}]^{T}$ .

Even though  $k_i$  is unknown, it suggests that after rotation and stretch, vector  $c_i$  can be converted to  $v_i$  and the relative relationships between elements of vector are constant, and it is just  $c_i \cong v_i$ .

So  $c_i$  can be used to stand for the partial mode shape of  $\lambda_i$ .

That is

$$\begin{bmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{m} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{21} & \cdots & c_{n1} \\ c_{12} & c_{22} & \cdots & c_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ c_{1m} & c_{2m} & \cdots & c_{n,m} \end{bmatrix} \begin{bmatrix} e^{\lambda_{1}t} \\ e^{\lambda_{2}t} \\ \vdots \\ e^{\lambda_{n}t} \end{bmatrix}$$
(14)

where  $c_{il}$  is the coefficient of  $e^{\lambda_i t}$  in the *l*-th state variable, column vector  $c_i \in C^{m \times 1}$  is the mode shape of  $\lambda_i$  from the PMP method. And just like Prony analysis,  $\lambda_i$  with the largest  $\|c_i\|_2$  is the dominant mode.

In practice, part of the mode shape composed of the largeamplitude state variable elements can maintain the main feature of the whole mode shape and is accurate for the mode shape matching method.

#### III. MODAL ANALYSIS

#### A. Power System Model

The power system model can be represented by a set of differential and algebraic equations.

$$\begin{cases} \dot{x} = f(x, y) \\ \mathbf{0} = g(x, y) \end{cases}$$
(15)

where  $\mathbf{x} \in \mathbb{R}^n$  and  $\mathbf{y} \in \mathbb{R}^m$  are state and algebraic variables, respectively.

Suppose power system is currently running around an operation point  $(x_0, y_0)$ . (15) can be linearized as

$$\begin{cases} \Delta \dot{\mathbf{x}} = \tilde{A} \Delta \mathbf{x} + \tilde{B} \Delta \mathbf{y} \\ \mathbf{0} = \tilde{C} \Delta \mathbf{x} + \tilde{D} \Delta \mathbf{y} \end{cases}.$$
(16)

Eliminating  $\Delta y$  in (16) gives

$$\dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x} \,, \tag{17}$$

where  $A = \tilde{A} - \tilde{B}\tilde{D}^{-1}\tilde{C}$  is the state matrix.

Let  $\lambda_i$  and  $v_i$ ,  $i = 1, 2, \dots, n$ , denote the *i*-th eigenvalues and corresponding eigenvectors of A, i.e.,

$$A\mathbf{v}_i = \lambda_i \mathbf{v}_i \tag{18}$$

Around such equilibrium point, after disturbance, the time domain responds of (17) can be represented as

$$\Delta \mathbf{x} = \sum_{i=1}^{n} k_i e^{\lambda_i t} \mathbf{v}_i \tag{19}$$

For the *i*-th state variable, that is,

$$\Delta x_i(t) = k_1 v_{i,1} e^{\lambda_i t} + k_2 v_{i,2} e^{\lambda_2 t} + \dots + k_n v_{i,n} e^{\lambda_n t} .$$
(20)

In (19) and (20),  $k_i$  is a constant.  $v_{i,j}$  in (20) denotes the *i*-th element of  $v_j$ . Obviously, the time domain response of  $\Delta x_i$  is related to all modes of the system.

#### B. Selection of Candidate Modes

In the proposed formula, after obtaining the dominant mode  $\lambda_m$  of the real time oscillation, Arnoldi method using the shiftinverse and Cayley transformation is applied to search the similar modes around  $\lambda_m$  in modal analysis, and the corresponding mode shapes can be also obtained [16].

Among these similar modes of  $\lambda_m$ , candidate modes for mode shape matching should be selected by mode comparison.

Here we define the mode error as

$$ME = \left| \frac{\lambda_{modal} - \lambda_m}{\lambda_m} \right|$$
(21)

where  $\lambda_{modal}$  is the mode from modal analysis.

Then the candidate modes should satisfy 2 requirements: *ME* with  $\lambda_m$  is smaller than the mode error tolerance  $\varepsilon_1$ , and damping ratio is smaller than the damping ratio tolerance  $\varepsilon_2$ .

#### IV. MODE SHAPE MATCHING METHOD

In a large-scale power system where the mode distribution is quite dense, differences between similar modes are too small. The dominant mode from the PMP method may correspond to one of these similar modes or may be a superposition of them. Considering differences between the mode shapes of these similar modes, we propose the mode shape matching method to solve this problem.

Here we use mode shapes of the candidate modes as the basis to fit that from the PMP method so that we can find the activated modes by their fitting coefficients.

That is

$$\boldsymbol{V} \cdot \boldsymbol{b} = \boldsymbol{c} \tag{22}$$

where  $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \dots, \boldsymbol{v}_s]$ ,  $\boldsymbol{v}_i \in \mathbb{C}^m$  is the mode shape of *i*-th candidate mode from modal analysis,  $\boldsymbol{c} \in \mathbb{C}^m$  is the mode shape of the dominant mode from the PMP method,  $\boldsymbol{b} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \dots, \boldsymbol{b}_s]^{\mathrm{T}}$ ,  $\boldsymbol{b}_i$  is the fitting coefficient of  $\lambda_i$ , and *s* is the number of fitting basis. Then  $\boldsymbol{b}$  will be solved with the least-square method.

The fitting result  $v_{fit}$  is

$$\boldsymbol{v}_{fit} = \boldsymbol{v} \cdot \boldsymbol{b} \tag{23}$$

where  $v = [v_1, v_2, \dots, v_s], b = [b_1, b_2, \dots, b_s]^T$ .

Define the match error of amplitude and angle as

$$ME_{amp} = \frac{\left\|abs(\mathbf{v}_{fit}) - abs(\mathbf{c})\right\|_{2}}{\left\|\mathbf{c}\right\|_{2}} ME_{angle} = \frac{sum\left|angle(\mathbf{v}_{fit} - \mathbf{c})\right|}{m}$$
(24)

If  $ME_{amp} < 2\%$  and  $ME_{angle} < 5^{\circ}$ , we think the mode shape matching is successful and obviously  $v_i$  with the largest  $b_i$  corresponds to the dominant mode  $\lambda_i$ .

#### V. SIMULATION STUDY

Simulations are used to evaluate the performance of the proposed online mode identification formula. The China Southern Power Grid is selected to validate the applicability in large-scale power system of the proposed mode shape matching method.

#### A. Implementation of the Algorithm

### 1) Online Estimation Using the PMP Method

Analyze the measured signals by the PMP method, and obtain the oscillation dominant modes and corresponding mode shapes. In Prony analysis, the problem of model order selection is still under research [5], [15]. In this paper, considering the relationship between sampling frequency and that of the oscillation, we define n=20 and then N=40 in (1).

## 2) Selection of the Candidate Modes

Obtain the closest modes of  $\lambda_m$  by Arnoldi method using shift-inverse and Cayley transformation, and then pick out the candidate modes by mode comparison. Considering the mode distribution of test system,  $\varepsilon_1$  and  $\varepsilon_2$  are set as 15% and 8% respectively after numerous simulations.

## 3) Mode Shape Matching

To explain in detail, if the mode from the PMP method is similar to more than one candidate mode in modal analysis, the candidate modes are chosen as the basis for the mode shape matching method. If the  $ME_{amp} < 2\%$  and  $ME_{angle} < 5^{\circ}$ , we think the fitting is success and the modes with larger fitting coefficient are dominating the oscillation.

#### B. China Southern Power Grid

The China Southern Power Gird is introduced here as an example of the large-scale power system. It is an AC/DC hybrid power system consisting of Guangzhou, Guangxi, Yunnan, Guizhou and Hainan five provinces. There are 12271 nodes, 13991 branches and 22 DC links. Rightmost modes distribution of this system is shown in Fig. 2.



Fig. 2. Rightmost modes distribution of China Southern Power Grid.

From Fig. 2 we can see that the mode distribution is so dense that many low-damping modes are quite close to each

other, which makes the mode shape matching necessary. In this test system, the measured state variables we select are the rotational speed difference  $\Delta \omega$  of these generators: DAHUAG1~5, ZEL1,2,4, 4DFDCg1~4, HESACG3, DONGFG1~4, MAW1~6. The oscillation caused by these low-damping modes can be observed through these large-amplitude state variables.

Assuming the system is operating in a certain equilibrium point, a three-phase short-circuit grounding fault occurs in Shuixia side of the tie line between Shuixia and Zengche. The fault will be removed after 5 periods. The proposed online mode identification formula is carried out as follow:

**Step 1**: The PMP method is applied to the measured ringdown data. In the results, we find two dominant modes  $-0.0647 \pm 1.2611i \pmod{1}$  and  $-0.1167 \pm 2.3894i \pmod{2}$ .

**Step 2**: The Arnoldi method is applied to search the closest modes in modal analysis of mode1 and mode 2 respectively, as shown in Fig. 3. By mode comparison, mode 1 only has one candidate mode  $-0.065 \pm 1.264i$ . As to mode 2, there are five candidate modes in modal analysis. The corresponding mode shapes are also obtained by modal analysis.



Fig. 3. The mode comparison of the China Southern Power Gird.

**Step 3.1**: Since there is no other mode whose *ME* with mode 1 is smaller than 0.15, we directly match the mode shape of  $-0.065 \pm 1.264i$  with that of mode 1. The *ME<sub>amp</sub>* and *ME<sub>angle</sub>* of mode 1 are 0.12% and 1.22°, which means the fitting result is exact enough.

**Step 3.2**: As to mode 2, there are five candidate modes in modal analysis similar to it as shown in Table. III. The mode shape from the PMP method of mode 2 is shown in Fig. 4 a). Then in (22), v consists of the mode shapes of five candidate modes shown in Table. I and c is the mode shape of mode 2 from the PMP method. Next, b is solved by (22) and shown in

Table. I. The fitting result  $v_{fit}$  is shown in Fig. 4 b). The *ME*<sub>amp</sub> and *ME*<sub>angle</sub> of mode 2 are 0.14% and 1.94 °, which means the fitting result is exact enough.

By the fitting coefficients, mode 2 -0.1167 $\pm$ 2.3894i from the PMP method is mainly composed of the modes - 0.111 $\pm$ 2.413i and -0.131 $\pm$ 2.610i from modal analysis.



Fig. 4. a) Mode shape of mode 2 -0.1167 $\pm$ 2.3894i from the PMP method; b) fitting result  $v_{fit}$  from the mode shape matching method.

In conclusion, by the proposed online mode identification formula,  $-0.065 \pm 1.264i$ ,  $-0.111 \pm 2.413i$  and  $-0.131 \pm 2.610i$  are the dominant modes of this oscillation, based on which the control actions should be taken.

 TABLE I

 FITTING COEFFICIENTS OF MODE SHAPE MATCHING

Candidate Mode	Mode Error	Damping Ratio	Amplitude of $\boldsymbol{b}$ (×10 <sup>-2</sup> )
$-0.089 \pm 2.042i$	0.14	4.35%	1.85
-0.107±2.365i	0.01	4.52%	1.11
-0.111±2.413i	0.01	4.60%	6.31
-0.131±2.610i	0.09	5.01%	2.36
-0.204±2.702i	0.14	7.53%	0.27

## VI. CONCLUSION

A novel online mode identification formula to find out the activated modes in the real-time oscillation of large-scale power system is proposed. It firstly applies the results from modal analysis to online identification and so overcomes the defect of measurement-based methods that results from measurement-based methods cannot be directly used to emergency oscillation control. The correctness and applicability of the proposed formal and methods has been evaluated on the China Southern Power Gird.

## VII. REFERENCES

 D. N. Kosterev, C. W. Taylor and W. A. Mittelstadt, "Model validation for the August 10, 1996 WSCC system outage," in *IEEE Trans. Power Syst.*, vol. 14, no. 3, pp. 967-979, Aug 1999.

- [2] R. T. Byerly, R. J. Bennon and D. E. Sherman, "Eigenvalue Analysis of Synchronizing Power Flow Oscillations in Large Electric Power Systems," in *IEEE Trans. Power Apps. Syst.*, vol. PAS-101, no. 1, pp. 235-243, Jan. 1982.
- [3] J. Rommes, N. Martins and F. D. Freitas, "Computing Rightmost Eigenvalues for Small-Signal Stability Assessment of Large-Scale Power Systems," in *IEEE Trans. Power Syst.*, vol. 25, no. 2, pp. 929-938, May 2010.
- [4] P. Kundur, N. Balu, and M. Lauby, *Power system stability and control*, ser. The EPRI power system engineering series. McGraw-Hill, 1994.
- [5] J. F. Hauer, C. J. Demeure and L. L. Scharf, "Initial results in Prony analysis of power system response signals," in *IEEE Trans. Power Syst.*, vol. 5, no. 1, pp. 80-89, Feb 1990.
- [6] M. L. Crow and A. Singh, "The matrix pencil for power system modal extraction," in *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 501-502, Feb. 2005.
- [7] N. Zhou, J. W. Pierre and J. F. Hauer, "Initial results in power system identification from injected probing signals using a subspace method," in *IEEE Trans. Power Syst.*, vol. 21, no. 3, pp. 1296-1302, Aug. 2006.
- [8] N. Zhou, J. W. Pierre, D. J. Trudnowski and R. T. Guttromson, "Robust RLS Methods for Online Estimation of Power System Electromechanical Modes," in *IEEE Trans. Power Syst.*, vol. 22, no. 3, pp. 1240-1249, Aug. 2007.
- [9] N. Zhou, D. J. Trudnowski, J. W. Pierre and W. A. Mittelstadt, "Electromechanical Mode Online Estimation Using Regularized Robust RLS Methods," in *IEEE Trans. Power Syst.*, vol. 23, no. 4, pp. 1670-1680, Nov. 2008.
- [10] L. Kumar and N. Kishor, "Determination of mode shapes in PMU signals using two-stage mode decomposition and spectral analysis," in *IET Gener. Transm. Distrib.*, vol. 11, no. 18, pp. 4422-4429, 12 21 2017.
- [11] D. J. Trudnowski, "Estimating Electromechanical Mode Shape From Synchrophasor Measurements," in *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1188-1195, Aug. 2008.
- [12] L. Dosiek and J. W. Pierre, "Estimating electromechanical modes and mode shapes using the multichannel ARMAX model," in *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 1950-1959, May 2013.
- [13] N. R. Chaudhuri and B. Chaudhuri, "Damping and Relative Mode-Shape Estimation in Near Real-Time Through Phasor Approach," in *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 364-373, Feb. 2011.
- [14] C. Li, H. D. Chiang and Z. Du, "Network-Preserving Sensitivity-Based Generation Rescheduling for Suppressing Power System Oscillations," in *IEEE Trans. Power Syst.*, vol. 32, no. 5, pp. 3824-3832, Sept. 2017.
- [15] N. Zhou, Zhenyu Huang, F. Tuffner, J. Pierre and Shuangshuang Jin, "Automatic implementation of Prony analysis for electromechanical mode identification from phasor measurements," *IEEE PES General Meeting*, Minneapolis, MN, 2010, pp. 1-8.
- [16] G. Angelidis and A. Semlyen, "Improved methodologies for the calculation of critical eigenvalues in small signal stability analysis," in *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1209-1217, Aug 1996.

**Chengfei Wang** is a postgraduate at Xi'an Jiaotong University. His fields of interest include power system stability and online identification.

**Zhengchun Du** was born in Shaanxi, China, in 1963. He received B.S., M.S., and Ph.D. degrees in electrical engineering from Xi'an Jiaotong University, Xi'an, China in 1983, 1986, and 1993, respectively. He is currently a professor of electrical engineering at Xi'an Jiaotong University. His research interests include power system stability and control.